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New performance bounds of a class of irreversible refrigerators

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Abstract. A general description is developed for a class of irreversible refrigerators operating between two heat reservoirs. This description is applied to refrigerators having thermal resistances between the working fluid of refrigerators and the heat reservoirs, heat leaks between the heat reservoirs, and internal dissipations of the working fluid in order to obtain the maximum performance coefficient and the performance coefficient versus cooling rate characteristics of such refrigerators. This description is also applied to optimizing the key performance parameters of such refrigerators. Some new performance bounds of of refrigerators are determined.

1. Introduction

Although there are many differences among conventional types of refrigerators, it is possible to place upper bounds on their performance via relatively simple thermodynamic models. One upper bound of practical interest is the maximum performance coefficient of refrigerators. According to classical thermodynamics, a reversible Carnot refrigeration cycle is the optimal configuration of conventional refrigerators operating between the heat reservoirs at temperatures T_h and T_c, and its performance coefficient is given by

$$\epsilon_c = T_c/(T_h - T_c). \tag{1}$$

However, the performance coefficient ϵ_c is of very limited practical value since it corresponds to reversible operation, i.e. infinitely slow operation and thus zero cooling rate. No practical engineer wants to design or build a refrigerator that runs infinitely slowly without producing the cooling rate. Therefore, it is necessary and significant to determine a new upper bound for the performance coefficient of refrigerators by using finite-time thermodynamics.

It is well known that the endoreversible cycle models [1–17] have played an important role in the development of finite time thermodynamics. But, using the endoreversible refrigeration cycle models of considering only finite-rate heat transfer between the working fluid of refrigerators and the heat reservoirs, one cannot obtain a new upper bound for the performance coefficient of refrigerators, so that it is necessary to establish a new irreversible refrigeration cycle model. On the other hand, like finite-time thermodynamic analysis of heat engines [18–24], it is also necessary for the investigation of refrigerators to develop several irreversible cycle models including various loss mechanisms, such as mechanical friction,

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heat leak, thermal resistance at the boundaries, and internal dissipation of the working fluid. For this purpose, we will first extend the endoreversible refrigeration cycle models to the irreversible refrigeration cycle model, which includes three major irreversibilities often existing in real refrigerators, and use it to analyse the optimal performance of a class of refrigerators operating between the heat reservoirs at temperatures T_h and T_c.

2. A new cycle model

For real refrigerators, besides the irreversibility of finite-rate heat transfer, there are also other sources of irreversibility, such as the heat leak between the heat reservoirs and the internal dissipation of the working fluid. In order to expound the influence of these irreversibilities on the performance of refrigerators, we consider a refrigeration system shown schematically in figure 1, where Q_1 and Q_2 are, respectively, the heats released to the hot reservoir at constant temperature T_h and absorbed from the cold reservoir at constant temperature T_c by the working fluid per cycle, Q_L is the heat leaking from the hot reservoir to the cold reservoir per cycle, T_1 and T_2 are, respectively, the temperatures of the working fluid in two isothermal processes, and W is the work input of the refrigerator per cycle.

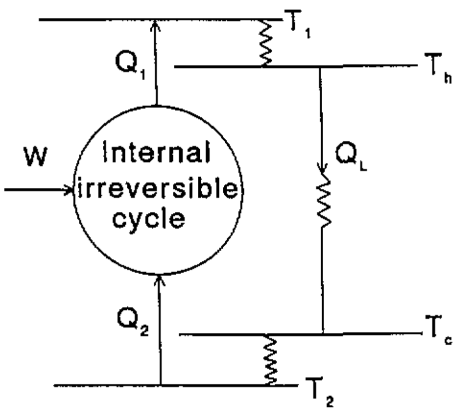

Figure 1. Schematic diagram of a refrigeration system.

The refrigerator operates in a cyclic fashion with fixed time t allotted for each cycle. The cycle of the working fluid is composed of two isothermal and two adiabatic processes. After time t has elasped, the working fluid returns to its initial state. When heat transfer obeys a linear law [5,7,21,25], Q_1, Q_2, and Q_L may, respectively, be expressed as

$$Q_1 = k_1(T_1 - T_h)t_1 \tag{2}$$

$$Q_2 = k_2(T_c - T_2)t_2 \tag{3}$$

$$Q_L = k_L(T_h - T_c)t \tag{4}$$

where t_1 and t_2 are, respectively, the times of two isothermal processes at temperatures T_1 and T_2, k_1 and k_2 are, respectively, the thermal conductances between the working fluid and two heat reservoirs at temperatures T_h and T_c, and k_L is the coefficient of the heat leak between the two heat reservoirs. According to figure 1, the net heats Q_h and Q_c transferred to the hot reservoir and from the cold reservoir per cycle are

$$Q_h = Q_1 - Q_L = k_1(T_1 - T_h)t_1 - k_L(T_h - T_c)t \tag{5}$$

and

$$Q_c = Q_2 - Q_L = k_2(T_c - T_2)t_2 - k_L(T_h - T_c)t \tag{6}$$

respectively.

To obtain the simple expressions of the performance coefficient and cooling rate, two adiabatic processes are often assumed to proceed in negligible time [21, 22, 26–28], such that the cycle time may be approximately given by

$$t = t_1 + t_2 \,. \tag{7}$$

Owing to the internal dissipation of the working fluid, all processes are irreversible. The entropy of the working fluid in two adiabatic processes increases. The TS diagram of the cycle differs from that of an endoreversible Carnot refrigeration cycle, as shown in figure 2, where ΔS_1 and ΔS_2 are, respectively, the entropy differences of the working fluid in two isothermal processes at temperatures T_1 and T_2, and they are defined as positive. According to the second law of thermodynamics, one has

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} > 0. \tag{8}$$

In order to obtain the quantitative relationship amongst the parameters Q_1, Q_2, T_1, and T_2, we introduce a new parameter

$$I = \Delta S_1/\Delta S_2 \tag{9}$$

such that the inequality in equation (8) can be written as

$$\frac{Q_1}{T_1} - I\frac{Q_2}{T_2} = 0. \tag{10}$$

It is seen clearly from equations (9), (10), and figure 2 that when $I = 1$, two adiabatic processes are reversible and the refrigeration cycle is endoreversible; when $I > 1$, two

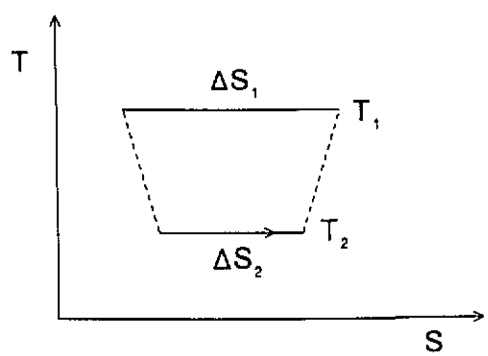

Figure 2. The TS diagram of an irreversible refrigeration cycle.

adiabatic processes are irreversible and the refrigeration cycle is internally irreversible. Thus, the parameter I is a measure of irreversibility in the two adiabatic processes. From equations (2)–(7), (10) and the definitions of the performance coefficient and cooling rate, we find that the expressions of the performance coefficient ϵ and cooling rate R are given by

$$\epsilon = \frac{Q_c}{W} = \frac{1 - Q_L/Q_2}{Q_1/Q_2 - 1} = \frac{1 - k_L(T_h - T_c)\left[\dfrac{1}{k_2(T_c - T_2)} + \dfrac{IX}{k_1(XT_2 - T_h)}\right]}{IX - 1} \tag{11}$$

and

$$R = \frac{Q_c}{t} = \frac{1}{\dfrac{1}{k_2(T_c - T_2)} + \dfrac{IX}{k_1(XT_2 - T_h)}} - k_L(T_h - T_c) \tag{12}$$

respectively, where $X = T_1/T_2$. Starting from equations (11) and (12), we can determine a new upper bound for the performance coefficient of a class of irreversible refrigerators and obtain the performance coefficient versus cooling rate characteristics of such refrigerators.

3. The maximum performance coefficient and the corresponding cooling rate

We now optimize the performance coefficient ϵ for two cases of $k_L = 0$ and $k_L > 0$.

(i) When $k_L = 0$, i.e. the heat leak between the heat reservoirs is negligible, $\epsilon = 1/(IX - 1)$ and $d\epsilon/dX < 0$. This implies the fact that ϵ is a monotonically decreasing function of X so that the maximum performance coefficient occurs at the boundary of the accepted X-range. When $X = T_h/T_c$, the maximum performance coefficient

$$\epsilon_{\max,I} = T_c/(IT_h - T_c)\,. \tag{13}$$

Like ϵ_c, $\epsilon_{\max,I}$ is also of limited practical value since it still corresponds to zero cooling rate. This shows that even though the influence of both finite-rate heat transfer and internal irreversibility on the performance of refrigerators is considered, one can not obtain a new practical significant upper bound for the performance coefficient of refrigerators. At the same time, equation (13) also shows the fact that the performance coefficient of all real refrigerators is always smaller than $\epsilon_{\max,I}$ so long as there is the internal irreversibility of the working fluid.

(ii) When $k_L > 0$, ϵ is not a monotonic function of X. This implies the fact that the maximum performance coefficient occurs somewhere in the accepted X-range. Using (11) and the extremal conditions:

$$\frac{\partial \epsilon}{\partial T_2} = 0 \qquad \frac{\partial \epsilon}{\partial X} = 0 \tag{14}$$

we find that when ϵ attains its maximum, T_2 and X have to satisfy the following equations:

$$T_2 = \frac{CT_c + T_h/X}{1 + C} \tag{15}$$

$$X = \frac{T_h}{T_c}\frac{1 + b\sqrt{d + (1 - d)/b}}{1 - b} \tag{16}$$

where $C = (Ik_2/k_1)^{1/2}$, $b = k_L(T_h - T_c)(1 + C)^2/(k_2T_c)$ and $d = T_c/(IT_h)$. Substituting (15) and (16) into (11), we find the maximum performance coefficient

$$\epsilon_{max} = \left(\frac{1-b}{\sqrt{b/d} + \sqrt{b - 1 + 1/d}}\right)^2 \tag{17}$$

of a class of refrigerators operating between the heat reservoirs at temperatures T_h and T_c and having three irreversibilities mentioned above. The corresponding cooling rate

$$R_m = k_L(T_h - T_c)\frac{(1-b)\sqrt{d + (1-d)/b}}{1 + b\sqrt{d + (1-d)/b}} \tag{18}$$

can be derived from (12), (15), and (16). ϵ_{max} is an important performance parameter of refrigerators, because it determines a new significant upper bound for the performance coefficient of refrigerators. Using (17), we can easily generate the curves of the maximum performance coefficient varying with k_L/k_2, as shown in figure 3. It is seen clearly from figure 3 that the maximum performance coefficient of refrigerators is sensitive to the parameter k_L as well as the parameter I.

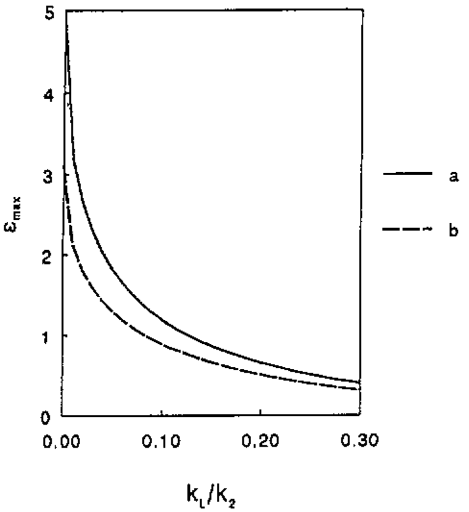

Figure 3. The curves of the maximum performance coefficient varying with k_L/k_2. Plots are presented for $T_h/T_c = 1.2$ and $k_1/k_2 = 1$. Curves (*a*) and (*b*) correspond to the cases of $I = 1$ and $I = 1.1$, respectively.

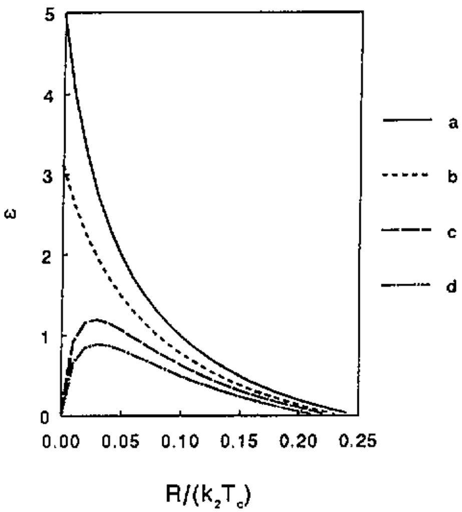

Figure 4. The curves of the performance coefficient ϵ varying with $R/(k_2T_c)$. The values of T_h/T_c and k_1/k_2 are the same as in figure 3. Curves (*a*), (*b*), (*c*) and (*d*) correspond to the cases of $I = 1$ and $k_L = 0$, $I = 1.1$ and $k_L = 0$, $I = 1$ and $k_L/k_2 = 0.1$, and $I = 1.1$ and $k_L/k_2 = 0.1$, respectively.

4. The ϵ–R characteristics

From (11), (12), and (15), we obtain

$$\epsilon = \frac{\left(1 - b - \frac{R}{kT_c}\right)\frac{R}{kT_c}}{\left(b + \frac{R}{kT_c}\right)\left(b + \frac{R}{kT_c} + \frac{1}{d} - 1\right)} \tag{19}$$

where $k = k_2/(1+C)^2$. Equation (19) determines the relationship between the performance coefficient and the cooling rate of a class of irreversible refrigerators operating between the heat reservoirs at temperatures T_h and T_c. Using the relationship, we can easily generate the curves of the performance coefficient varying with the cooling rate of refrigerators, as shown in figure 4. Curves (c) and (d) in figure 4 correspond to the case of $k_L/k_2 = 0.1$. The two curves indicate that when the heat leak between the heat reservoirs is considered, there exists a maximum performance coefficient with non-zero cooling rate. The expressions of the maximum performance coefficient and the corresponding cooling rate may be derived from (19). The results are the same as (17) and (18), respectively. Curves (c) and (d) also indicate that when $\epsilon < \epsilon_{max}$, there are two different R for a given ϵ, where one is smaller than R_m and the other is larger than R_m. When $R < R_m$, the performance coefficient ϵ decreases as the cooling rate R decreases, such that the working states of $R < R_m$ are not the optimal operating states of refrigerators, that is to say, the rational region of the cooling rate of refrigerators should be

$$R \geqslant R_m \,. \tag{20}$$

This implies that R_m is also an important performance parameter of refrigerators. It determines a lower bound for the cooling rate of refrigerators.

5. Optimal analysis of other performance parameters

(i) When the refrigerator operates in the state of maximum performance coefficient, the temperatures of the working fluid in two isothermal processes

$$T_{1m} = T_h \frac{1 - b + C + bC\sqrt{d + (1-d)/b}}{(1+C)(1-b)} \tag{21}$$

and

$$T_{2m} = T_c \frac{1 - b + C + bC\sqrt{d + (1-d)/b}}{(1+C)[1 + b\sqrt{d + (1-d)/b}} \tag{22}$$

can be derived from (15) and (16). According to (20), we can determine that rational regions of the temperatures of the working fluid in two isothermal processes should be

$$T_1 \geqslant T_{1m} \tag{23}$$

and

$$T_2 \leqslant T_{2m} \,. \tag{24}$$

Equations (23) and (24) determine new bounds for the temperatures of the working fluid in two isothermal processes.

(ii) Using (10) and (15), we can prove that the optimal ratio of the times spent on two isothermal processes is given by

$$\frac{t_1}{t_2} = \sqrt{\frac{I k_2}{k_1}} \,. \tag{25}$$

Equation (25) shows that the optimal ratio of the times spent on two isothermal processes is not affected by the heat leak, but it depends on the parameters k_2/k_1 and I.

(iii) When the refrigerator operates in the state of the maximum performance coefficient, the power input that it requires

$$P_{\mathrm{m}} = k_{\mathrm{L}}(T_{\mathrm{h}} - T_{\mathrm{c}})\frac{1}{d}\frac{b}{1-b}\left[1+\sqrt{d+(1-d)/b}\right]^2 \frac{\sqrt{d+(1-d)/b}}{1+b\sqrt{d+(1-d)/b}} \tag{26}$$

can be calculated from (17) and (18). Equation (26) determines a lower bound for the power input of refrigerators, because the refrigerator does not operate in the rational region of $R \geqslant R_{\mathrm{m}}$ when $P < P_{\mathrm{m}}$.

6. Conclusions

The important feature of the irreversible refrigeration cycle model adopted in this paper is that it can include three major irreversibilities often existing in real refrigerators and it is still a simpler model which has analytic solutions. The maximum performance coefficient with non-zero cooling rate and other new performance bounds related to important performance parameters of refrigerators are determined. These results are general and useful. They can serve as an excellent guide to the evaluation of existing refrigerators and the optimal design of future refrigerators.

References

[1] Curzon F L and Ahlborn B 1975 *Am. J. Phys.* **43** 22
[2] Andresen B 1983 *Finite Time Thermodynamics* (Copenhagen: University of Copenhagen)
[3] Sieniutycz S and Salamon P 1991 Finite Time Thermodynamics and Thermoeconomics *Advances in Thermodynamics* vol 4 (London: Taylor & Francis)
[4] Bejan A 1988 *Advanced Engineering Thermodynamics* (New York: Wiley)
[5] Ondrechen M J, Rubin M H and Band 1983 *J. Chem. Phys.* **78** 4721
[6] De Vos A 1987 *J. Phys D: Appl. Phys.* **20** 232
[7] Chen J and Yan Z 1988 *J. Appl. Phys.* **63** 4795
[8] Gordon J M 1989 *Am. J. Phys.* **57** 1136
[9] Chen J and Yan Z 1989 *Phys. Rev.* A **39** 4140
[10] Chen L and Yan Z 1989 *J. Chem. Phys.* **90** 3740
[11] Wu C 1990 *Energy Convers. Mgmt* **30** 261
[12] Yan Z and Chen J 1990 *J. Phys. D: Appl. Phys.* **23** 136
[13] Agrawal D C and Menon V J 1990 *J. Phys. A: Math. Gen.* **23** 5319
[14] Wu C 1991 *Energy Convers. Mgmt* **31** 561
[15] Chen J 1992 *J. Appl. Phys.* **72** 3778
[16] Gordon J M and Orlov V N 1993 *J. Chem. Phys.* **74** 5303
[17] Chen J and Yan Z 1993 *J. Phys. D: Appl. Phys.* **26** 1581
[18] Andresen B, Salamon P and Berry R S 1977 *J. Chem. Phys.* **66** 1571
[19] Orlov V N and Berry R S 1990 *Phys. Rev.* A **42** 7203
[20] Gordon J M and Huleihil M 1990 *J. Appl. Phys.* **69** 1
[21] Gordon J M and Huleihil M 1992 *J. Appl. Phys.* **72** 829
[22] Wu C and Kiang R L 1992 *Energy* **17** 1173
[23] Orlov V N and Berry R S 1992 *Phys. Rev.* A **45** 7202
[24] Romm F 1993 *J. Appl. Phys.* **74** 5310
[25] Wu C 1991 *Energy* **16** 757
[26] Salamon P and Nitzan A 1981 *J. Chem. Phys.* **74** 3546
[27] Wu C 1988 *Energy Convers. Mgmt* **28** 207
[28] Yan Z and Chen J 1992 *Int. J. Energy Environment Economics* **2** 63